December 2019

MATHEMATICS

Time Allowed: 3 Hours *Full Marks: 70*

Answer to Question No.1 is compulsory and to be answered first.
This answer is to be made in separate loose script(s) provided for the purpose.
Maximum time allowed is 45 minutes, after which the loose answer scripts will be collected and fresh answer scripts for answering the remaining part of the question will be provided.
On early submission of answer scripts of Question No.1, a student will get the remaining script earlier.

Answer <u>any five</u> questions from Group-A, B & C, taking <u>at least one</u> from each group.

1. Answer <u>any twenty</u> questions with minimum justification: 20×1

i) $\log_7 3 \times \log_6 7 \times \log_5 6 \times \log_4 5 \times \log_3 4 =$ (a) 0 (b) 1 (c) 2 (d) none of these.

ii) If $\log_e 2 \times \log_x 625 = \log_{10} 16 \times \log_2 10$ then $x =$ (a) 4 (b) 5 (c) 1/5 (d) none.

iii) If one root of $3x^2 + px + 3 = 0$ is square of the other then $p =$ (a) $\frac{2}{3}$ (b) 3 (c) 1 (d) $\frac{1}{2}$

iv) If α, β are the roots of $x^2 - px + q = 0$ then the value of $\alpha^2 + \beta^2 + \alpha\beta$ is – (a) $q^2 - p$ (b) $q - p^2$ (c) $p^2 - q$ (d) $p - q^2$

v) A right pyramid of height 15cm stands on a square base of side 16cm. its volume is – (a) 1260cc. (b) 1280cc. (c) 1020cc. (d) None of these.

vi) Which of the following fractions are proper fractions – (a) $\frac{x^4+x^3}{x^2+1}$ (b) $\frac{2x^3+x^2+1}{x^3-1}$ (c) $\frac{x^2+7}{3x^3+x+x^2}$ (d) None of these.

vii) The value of $\vec{j} \times (\vec{k} \times \vec{i})$ is – (a) 1 (b) $\vec{i}$ (c) 0 (d) $\vec{k}$

viii) The term independent of x in the expansion of $\left(1 - \frac{1}{x}\right)^{10}$ is – (a) 4th (b) 5th (c) 6th (d) none of these.

ix) The number of terms in the expansion of $\left(x^2 - 2 + \frac{1}{x^2}\right)^6$ is – (a) 6 (b) 7 (c) 5 (d) 13.

x) $(1 - 2x)^{-\frac{1}{2}}$ can be expanded in a binomial series if – (a) $x > \frac{1}{2}$ (b) $-2 < x < 2$ (c) $-\frac{1}{2} < x < \frac{1}{2}$ (d) $-1 < x < 1$

xi) If $\tan x \tan 3x = 1$ then the value of $\tan 2x$ is – (a) 2 (b) 1 (c) $-\sqrt{3}$ (d) none of these.

xii) The square root of $(3 + 4i)$ is – (a) $\sqrt{3} + i$ (b) $2 - i$ (c) $2 + i$ (d) none of these.

xiii) The angle between the vectors $(4\vec{i} + 2\vec{j} - 7\vec{k})$ and $(3\vec{i} + 2\vec{j} + 2\vec{k})$ is (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these.

xiv) If $\bar{z}$ is conjugate to z then $\text{amp}(z) + \text{amp}(\bar{z}) =$ (a) 0 (b) π (c) 2π (d) None of these.

xv) If $10\alpha = \frac{\pi}{2}$, then $\tan 3\alpha \cdot \tan 5\alpha \cdot \tan 7\alpha =$ (a) 0 (b) 1 (c) 2 (d) none of these.

xvi) The value of $\sin\left[\tan^{-1}x + \tan^{-1}\frac{1}{x}\right]$ = (a) 0 (b) 1 (c) -1 (d) $\sqrt{2}$.

xvii) The value of $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0}$ is – (a) 1 (b) 2 (c) 3 (d) 4.

xviii) The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is – (a) 3/2 (b)3/10 (c) 6 (d) none of these.

xix) The centre of the circle $ax^2 + (2a - 3)y^2 - 4x - 1 = 0$ is – (a) $\left(\frac{2}{3}, 0\right)$ (b) $\left(-\frac{2}{3}, 0\right)$ (c) (2,0) (d) None of these.

xx) The vertex of the parabola $y^2 = 8x + 4y + 4$ is – (a) (-1,-2) (b) (2,1) (c) (1,-2) (d) (-1,2).

xxi) The unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is – (a) $\vec{a} \times \vec{b}$ (b) $\vec{b} \times \vec{a}$ (c) $\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$ (d) $\frac{\vec{a}\times\vec{b}}{|\vec{a}\cdot\vec{b}|}$

xxii) The area of a parallelogram, two adjacent sides of which are represented by $\vec{a}$ and $\vec{b}$ is – (a) $\vec{a} \times \vec{b}$ (b) $\frac{1}{2}(\vec{a} \times \vec{b})$ (c) $\vec{a}.\vec{b}$ (d) $\frac{1}{2}(\vec{a}.\vec{b})$ (e) None of these.

xxiii) The function $f(x) = \sqrt{1 + x + x^2} - \sqrt{1 - x + x^2}$ is – (a) even function (b) odd function (c) periodic function (d) parametric function.

xxiv) If $\varphi(x) = \log \sin x$ and $\Psi(x) = \log \cos x$ then $e^{2\phi(x)} + e^{2\Psi(x)}$ is – (a) 0 (b) 2 (c) 1 (d) None of these.

xxv) The value of $\lim_{x\to 0} \frac{\cos x}{\frac{\pi}{2} - x}$ is – (a) 1 (b) 0 (c) 2 (d) -1.

xxvi) The minimum value of $y = x^2 - x + 2$ is – (a) 7/4 (b) 7/2 (c) 7/5 (d) ¾.

xxvii) The function $f(x) = \sqrt{1 - x^2}$ is not defined for – (a) $x = 1$ (b) $x = 0$ (c) $|x| > 1$ (d) none of these.

xxviii) If $f(x) = \log_e e^x + e^{\log_e x}$ then $f'(x)$ = (a) 2 (b) $e^x + 1$ (c) $e^x + x$ (d) None of these.

Group-A

2. a) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$ then show that $x^{y+z}.y^{z+x}.z^{x+y} = 1$.

b) If the coefficient of x^7 in the expansion of $\left(px^2 + \frac{1}{qx}\right)^{11}$ be equal to the coefficient of x^{-7} in the expansion of $\left(px - \frac{1}{qx^2}\right)^{11}$ then prove that $pq = 1$. 5+5

3. a) If $x + \frac{1}{x} = 2\cos\theta$, $y + \frac{1}{y} = 2\cos\varphi$ then use De Moivre's theorem to show that— $x^3y^4 + \frac{1}{x^3y^4} = 2\cos(3\theta + 4\varphi)$.

b) If α be a root of $4x^2 + 2x - 1 = 0$ then prove that $(4\alpha^3 - 3\alpha)$ is the other root. 5+5

4. a) Prove that $(1 + x + x^2 + \cdots\infty)(1 - x + x^2 - \cdots\infty) = (1 + x^2 + x^4 + \cdots\infty)$

b) If $\hat{a}, \hat{b}$ are unit vectors and θ is the angle between them, then show that $\sin\frac{\theta}{2} = \frac{1}{2}|\hat{a} - \hat{b}|$ 5+5

5. a) Is the fraction $\frac{3x}{x^2+2x-8}$ proper or improper ? Split the fraction into partial fractions.

b) Find the unit vector perpendicular to both the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = 7\hat{i} - 5\hat{j} + \hat{k}$. 5+5

Group-B

6. a) Find the equation of the circle which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at (5,5) and whose radius is 5 units.

   b) A right prism of height 12cm. stands on a base which is a regular hexagon. If the area of the whole surface of the prism be $1152\sqrt{3}$sq.cm., find the volume of the prism. 5+5

7. a) If $\tan\frac{\theta}{2} = \tan^3\frac{\varphi}{2}$ and $\tan\varphi = 2\tan\alpha$, then show that $\theta + \varphi = 2\alpha$.

   b) If $2\tan\alpha = 3\tan\beta$, then show that $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$. 5+5

8. a) Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$

   b) Find the equation of the straight line which passes through the point of intersection of the lines $2x + 3y = 5$, $3x + 5y = 7$ and makes equal intercepts upon co-ordinate axes. 5+5

Group-C

9. a) A function is defined by $f(x) = \frac{|x|}{x}$, $x \neq 0$. Draw a rough sketch of the function. What value must be assigned to f(x)at $x = 0$, so that f(x) is continuous at $x = 0$.

   b) Evaluate $\lim_{x\to a}\frac{x\sin a - a\sin x}{x - a}$ (2+3)+5

10. a) i) If $y = \tan^{-1}\left[\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right]$ then find $\frac{dy}{dx}$

    ii) If $y = \sin x^0$, find $\frac{dy}{dx}$.

    b) If $y = \sin(m\sin^{-1}x)$, then prove that y satisfies $(1 - x^2)y_2 - xy_1 + m^2y = 0$ where suffixes denote differentiation with respect to x. (3+2)+5

11. a) An open tank with square base is to be made by metallic sheet of 48 sq.mt. What will be the dimensions of the tank so that it may contain maximum quantity of water?

    b) If the area of a circle increases at a uniform rate then show that the rate of increase of its perimeter varies inversely as the radius. 5+5
