

MATHEMATICS-II

Time Allowed: 2.5 Hours

Full Marks : 60

Answer the following questions from Group A, B & C as directed

Group A

1. Choose the correct alternatives(any ten) :

1×10

- i) The value of $\begin{vmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{vmatrix}$ is (a) 2 (b) 0 (c) -1 (d) none of these.
- ii) Determinant value of any skew symmetric matrix of order 2×2 is
(a) strictly positive (b) strictly negative (c) 0 (d) none of these.
- iii) The co-factor of the element 5 in $D = \begin{vmatrix} 1 & 3 & 4 \\ -1 & 0 & 2 \\ 3 & 5 & 8 \end{vmatrix}$ is (a) 5 (b) 6 (c) 0 (d) -6.
- iv) The value of k for which the points $(1, 2)$, $(k, 4)$ and $(3, 6)$ are collinear is
(a) 1 (b) 2 (c) 3 (d) none of these.
- v) The value of k for which the line $2x + 3y + 4 + k(6x - y + 11) = 0$ is parallel to the y -axis is
(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) 3 (d) none of these.
- vi) Coordinates of centre of the circle $2x^2 + 2y^2 - 8x - 5 = 0$ are
(a) $(0, 2)$ (b) $(2, 0)$ (c) $(4, 0)$ (d) $(-4, 0)$
- vii) $\int e^{1-\log x} dx$ is equal to (a) $ex + c$ (b) $e \log|x| + c$ (c) $x \log e + c$ (d) none of these.
- viii) $\int \frac{\cos x}{\sqrt{1 + \cos 2x}} dx =$ (a) $x + c$ (b) $\frac{x}{2} + c$ (c) $\frac{x}{\sqrt{2}} + c$ (d) none of these.
- ix) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^5 - x^3\left(\frac{dy}{dx}\right)^3 = 0$ are
(a) 1 and 2 (b) 2 and 5 (c) 2 and 3 (d) none of these.
- x) The Integrating Factor (IF) of the differential equation $\frac{dy}{dx} + \frac{1}{x}y = \sin x$ is
(a) x (b) $\frac{1}{x}$ (c) $\log_e x$ (d) none of these.
- xi) General solution of the differential equation $\frac{dy}{dx} = 1$ is
(a) $y = c$ (b) $x = 0$ (c) $y = x$ (d) none of these.
- xii) The differential equation whose general solution is $y = A \cos x + B \sin x$ is
(a) $\frac{d^2y}{dx^2} = y$ (b) $\frac{d^2y}{dx^2} = -y$ (c) $\frac{d^2y}{dx^2} = 0$ (d) none of these.
- xiii) If $u = 5x^2 + 7xy + 3y^2$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ equals to
(a) u (b) $2u$ (c) $4u$ (d) none of these.

- xiv) The median of the data 6, 4, -1, 0, 4, 8, 9 is
 (a) 0 (b) 4 (c) 6 (d) none of these.
- xv) Two coins are tossed. The probability of getting at least one head is
 (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) none of these.

1 × 10

2. Fill in the blanks(any ten) :

- i) For a matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 0 & 5 \end{pmatrix}$, $A^T =$ _____.
- ii) Matrix $(1 \ 2) \begin{pmatrix} -2 & 5 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is equal to _____.
- iii) The area bounded by the curves $4x + 3y = 12$, $y = 0$ and $x = 0$ is _____.
- iv) If the length of the diameter of the circle $x^2 + y^2 + 4x - 6y - k = 0$ is 4 then value of k is _____.
- v) $x^2 - y^2 = 2022$ represents a _____.
- vi) If $\frac{d}{dx}[\phi(x)] = f(x)$, then $\int f(x) dx$ is _____.
- vii) $\int_0^1 |x| dx =$ _____.
- viii) $\int_0^1 \frac{1-x}{1+x} dx =$ _____.
- ix) Particular Integral(PI) of the differential equation $\frac{d^2y}{dx^2} + 4y = 5e^{2x}$ is _____.
- x) If the differential equation $(2x^2 + 4y)dx + k(4x + y - 1)dy = 0$ is exact then value of k is _____.
- xi) The Complementary Function(CF) of $\frac{d^2y}{dx^2} - 16 = xe^{3x}$ is _____.
- xii) $y = c_1 + c_2e^x$ is the solution of the differential equation _____.
- xiii) If $u = e^{xy}$, then the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is _____.
- xiv) If the mean of $x - 2$, 10 , $x + 3$, 7 is 9 , then $x =$ _____.
- xv) The probability of getting an even number from the numbers up to 50(Including 50) is _____.

3. Answer the following questions(any ten) :

1 × 10

- i) Find x and y such that $\begin{pmatrix} 3 & x+1 \\ y+2 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}$.
- ii) What is symmetric matrix?
- iii) Find the value of x for which $A = \begin{pmatrix} 2 & x \\ 3 & 6 \end{pmatrix}$ is singular matrix.
- iv) Find slope of the line be $2x - 3y = 3$.
- v) Find length of major axis of the ellipse $9x^2 + 16y^2 = 144$.
- vi) Integrate : $\int \cot^2 x dx$.

vii) Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^9 x \cos^5 x \, dx$.

viii) Evaluate: $\int_1^2 \log x \, dx$.

ix) If the roots of Auxilliary Equation(AE) of a linear differential equation of 2nd order with constant coefficients be $2 \pm 3i$, find the Complimentary Function(CF).

x) Find $\frac{1}{D^2 + 5}(\sin 2x)$.

xi) If $u = 7xy^2$, find $\frac{\partial^2 u}{\partial x \partial y}$.

xii) State Euler's theorem on homogeneous function.

xiii) If $y = 2x - 5$ and $\bar{x} = 7$, find the value of \bar{y} .

xiv) Find the mode of the data: 7, -1, 2, 3, 5, 0, 2, 3, 2, 3, 0, 4, 2, 5, 1

xv) Let A and B be two events and $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.4$, find the value of $P(A \cup B)$.

Group B

4. Answer the following questions(any six) :

2×6

i) If $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, find $A(\text{adj } A)$.

ii) Show that $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{vmatrix} = - \begin{vmatrix} \alpha^2 & \beta^2 & \gamma^2 \\ \alpha & \beta & \gamma \\ 1 & 1 & 1 \end{vmatrix}$

iii) Transfer the Cartesian form of the equation $x^2 + y^2 = 2ax$ to its polar form.

iv) If the lines joining the points (a, -2) and (4, -a) be parallel to the line joining (2, 1) and (3, 4), find the value of a.

v) Integrate: $\int \frac{dx}{1 - \cos x}$.

vi) Evaluate : $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} \, dx$.

vii) Solve : $x dy - y dx = (x^2 + y^2) dx$

viii) Verify Euler's Theorem for the function $f(x, y) = 3x^2 + 4y^2$.

ix) Find the variance of : 5, 6, 7, 8, 10.

x) An urn contains 12 white balls and 6 red balls. A ball is drawn at random. What is the probability of being it to be a white?

Group C

5. Answer the following question(any one) :

i)

a) If A be a skew symmetric matrix then prove that A^2 is symmetric. 2

b) Evaluate the determinant $\begin{vmatrix} 0 & -1 & -1 & 1 \\ 4 & 5 & 1 & 1 \\ 3 & 9 & 4 & 1 \\ 4 & 4 & 4 & 1 \end{vmatrix}$ using Chio's method. 4

ii)

a) Without expanding prove that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$. 2

b) If $A = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$, prove that $A^2 - 4A - I = 0$, where I is the unit matrix of order 2 and hence find A^{-1} . 4

iii) A straight line passes through the point (7, 3) and has intercepts on the axes in equal magnitude but opposite in sign. Find its equation. Also find the co-ordinates of the point on the line at which the abscissa is triple the ordinate. 6

6. Answer the following question(any one) :

i) Evaluate : a) $\int \frac{e^{4\log x} - e^{3\log x}}{e^{\log x} - 1} dx$ b) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1 + \sin x)(2 + \sin x)} dx$. 2 + 4

ii) Solve : $\frac{d^2y}{dx^2} - 4a \frac{dy}{dx} + 4a^2y = x^2e^{2ax}$. 6

iii) Mark the area bounded by $y^2 = 2x$, $x = 1$ and $x = 4$ and find the enclosed area. 6

7. Answer the following question(any one) :

i) A distribution consists of three components with frequencies 200, 250, 300 having means 25, 10 and 15; standard deviation 3, 4 and 5 respectively. Prove that the mean of the combined distribution is 16 and its standard deviation is 7.2 approximately. 6

ii) A coin and a die are thrown simultaneously. Find the probabilities of occurrence of
a) head and even face b) tail and multiple of three. 6

iii) If $u = \cos^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$. 6